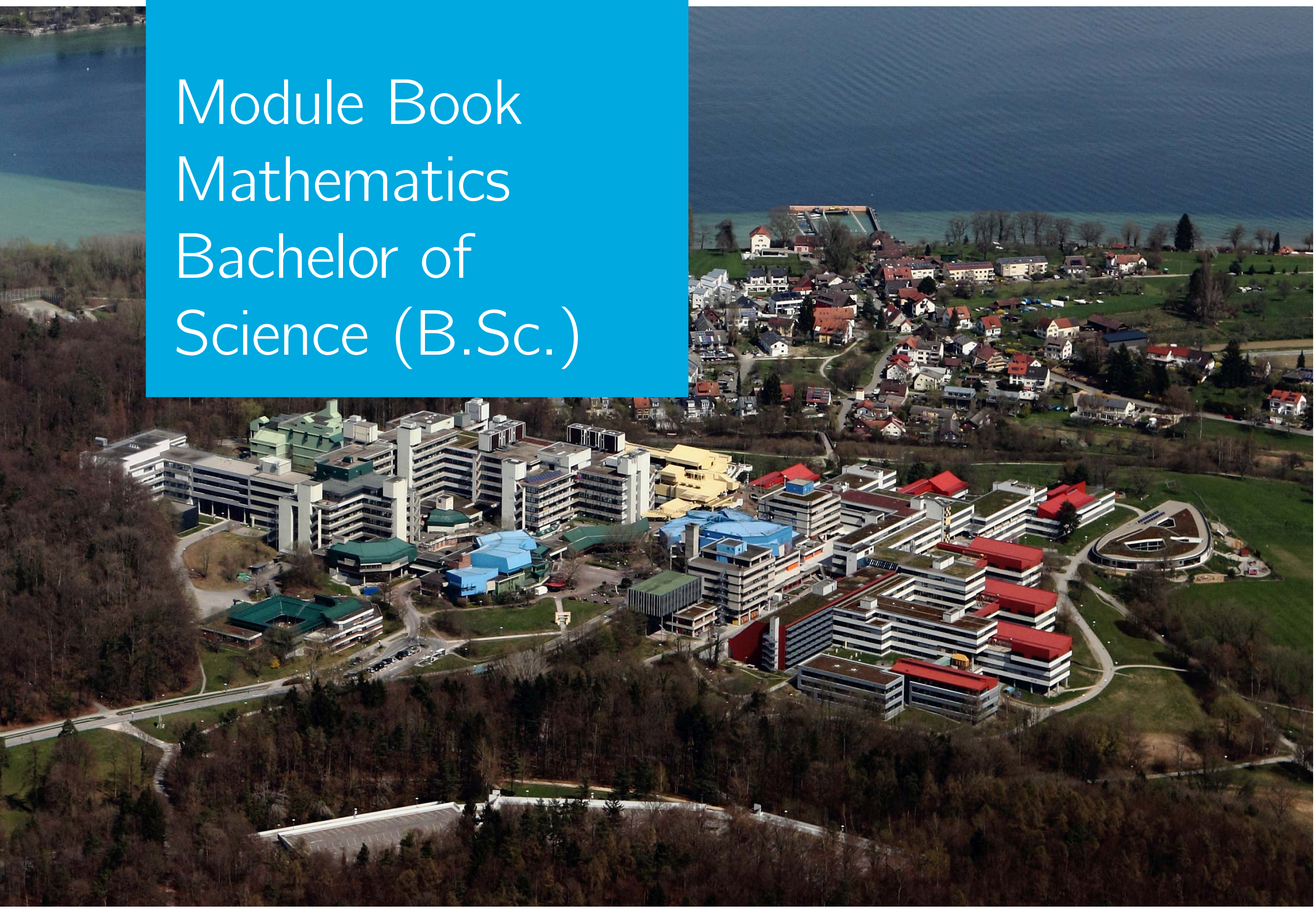
Universität Konstanz

Module Book Mathematics Bachelor of Science (B.Sc.)



Department of Mathematics and Statistics

As of January 2024

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1 Qualification Goals

The study of mathematics is a scientific education that provides the basis for a later career in various branches of business, industry or research. The main focus of the study programme is on learning mathematical theories and methods, the practical implementation and application of these methods and the ability to communicate this knowledge. In addition to imparting specific mathematical knowledge, specific ways of thinking and working are acquired, which are characterized by the ability to abstract, rigor, creativity and tenacity. As these skills are in demand in many areas of industry and business as well as at schools and universities and are also of social relevance, they represent an important goal, that is automatically imparted when dealing with mathematics. Through the intensive active engagement with mathematical content, students learn a flexibility and and openness of thought, coupled with rigor and self-criticism, which can also be extended to other areas of professional and public life. Through the active acquisition of well-founded mathematical knowledge, students acquire the ability to recognize analogies and basic patterns as well as the ability to recognize, formulate and solve complex problems. They practise conceptual, analytical and logical thinking and develop strategies for lifelong learning. The Bachelor's degree program Mathematics aims to provide a basic mathematical education. The graduates are able to understand and apply mathematical models in science and in business. In addition to the purely technical training, the students also develop the ability to analyze and solve problems, moreover communication skills and perseverance are strengthened. Students who wish to make the transition to professional life after their Bachelor's degree can organize their studies in such a way that they get to know the basic mathematical aspects of the intended professional field. On the other hand, it is of course also possible to lay a stronger scientific focus with regard to the subsequent Master's degree courses. The successfully completed Bachelor's degree program should, among other things enable

- to work in a team of mathematicians, computer scientists, natural scientists, engineers or economists in industry and business,
- to perform tasks in the areas of development, application and sales,
- to pursue further qualification in continuing education programs and
- to pursue a qualified Master's degree.

2 Compulsory modules

For all module units that have exercises as a component, successful participation in the exercises is a prerequisite for successful completion of the module unit. Successful participation in the exercises typically consists of 50% of the exercise points and active participation in the tutorial.

Compulsory module Analysis I/II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
18	2 semesters	12	Mandatory module

Module units:

- Analysis I
- Analysis II

Module grade: The sum of the better exam grade multiplied by 2/3 and the worse exam grade multiplied by 1/3.

Learning objectives: The objectives of the module include familiarity with fundamental topics in analysis such as proof techniques, knowledge of continuity, convergence, differentiability, integrals, etc. These are essential prerequisites for further study.

Learning outcomes: The students

- know and understand the fundamental concepts, statements, and methods of analysis,
- understand proof techniques,
- recognize the concept of the limit as fundamental to analysis,
- can assess which analytical tools are appropriate for various problem scenarios,
- can independently apply theorems and prove minor extensions on their own,
- possess a secure, precise, and independent command of the concepts, statements, and methods of analysis acquired during the exercises,
- recognize and utilize cross-connections to the other mandatory modules,
- have honed their ability to acquire knowledge through self-study and
- have enhanced competencies in presenting and communicating by delivering their own solutions during the tutorials

Module unit Analysis I

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lecture 4 SWS - tutorial 2 SWS	1st semester	annually (winter semester)	German

Recommended prerequisites: none

Teaching contents:

- sets, mappings, elements of logic, cardinality
- number systems: real numbers, complex numbers
- sequences, series, limits

- power series, uniform convergence
- elements of topology and functional analysis: metric spaces, compact sets/spaces
- continuity and differentiability in one variable

topics that are dealt with in Analysis I or Analysis II:
- continuity in multiple variables or in metric spaces
- metric spaces, connectedness, product spaces
- Riemann or Lebesgue integral, interchange of limit processes, transformation theorem
- Taylor series

Form of examination:
- written exam
- successful participation in the exercises

Work load: 270 h
- on-campus study (lectures and tutorials) 14 × 4,5 h = 63 h
- preparation and follow-up 14 × 3 h = 42 h
- exercises 14 × 9 h = 126 h
- exam preparation: 39 h

Module unit Analysis II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lectures 4 SWS - tutorials 2 SWS	2nd semester	annually (summer semester)	German

Recommended prerequisites: Analysis I, Linear Algebra I

Teaching contents:
- differentiability in multiple variables
- local invertibility, Banach's fixed-point theorem, theorem on implicit functions
- extrema under constraints
- multidimensional integration, Gauss's theorem

topics that are dealt with in Analysis I or Analysis II:
- Continuity in multiple variables or in metric spaces
- metric spaces, connectedness, product spaces
- Riemann or Lebesgue integral, interchange of limit processes, transformation theorem
- Taylor series

Form of examination:
- written exam
- successful participation in the exercises

Work load: 270 h
- on-campus study (lectures and tutorials) 14 × 4,5 h = 63 h

- preparation and follow-up 14×3 h = 42 h
- exercises 14×9 h = 126 h
- exam preparation: 39 h

Compulsory Module Linear algebra I/II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
18	2 semester	12	Compulsory Module

Module units:

- Linear algebra I
- Linear algebra II

Module grade: Sum of the better exam grade multiplied by 2/3 and the worse exam grade multiplied by 1/3.

Learning objectives: Familiarity with the theoretical and practical foundations, as well as the fundamental algorithms of linear algebra. Almost all parts of mathematics and its applications build upon these techniques. Therefore, this module is at the beginning of the curriculum.

Learning outcomes: The students

- understand basic abstract set-theoretical and algebraic structures and constructions,
- understand the axiomatic method and the principles of mathematical rigor,
- can apply abstract theorems and methods to concrete mathematical problems,
- analyze linear geometric situations using abstract algebraic and concrete computational methods,
- can independently prove simpler statements from linear algebra,
- can justify the correctness of more complex statements in linear algebra.

Module unit Linear algebra I

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lecture 4 SWS - tutorials 2 SWS	1st semester	annually (winter semester)	German

Recommended prerequisites: none

Teaching contents:

- Theoretical and practical foundations of linear algebra
- Sets and mappings, groups, rings, fields, vector spaces, matrix calculus, determinant, eigenvalues, characteristic polynomial, scalar products

Form of examination:

- exam
- successful participation in the exercises

Work load: 270 h

- on-campus study (lectures and tutorials)14 $\times$ 4,5 h = 63 h
- preparation and follow-up 14 $\times$ 3 h = 42 h
- exercise sheets 14 $\times$ 9 h = 126 h
- exam preparation: 39 h

Module unit Linear algebra II

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lecture 4 SWS - tutorial 2 SWS	2nd semester	annually (summer semester)	German

Recommended prerequisites: Linear algebra I

Teaching contents: Jordan Normal Form, bilinear and multilinear mappings, quadratic and alternating forms, Sylvester's signature, orthogonalization, orthogonal and unitary mappings, self-adjoint and normal mappings, Spectral Theorem

Form of examination:

- exam
- successful participation in the tutorials

Work load: 270 h

- on-campus studies (lectures and tutorials) 14 $\times$ 4,5 h = 63 h
- preparation and follow-up 14 $\times$ 3 h = 42 h
- exercise sheets 14 $\times$ 9 h = 126 h
- exam preparation: 39 h

Compulsory Module Practical mathematics I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
18	2 semesters	12	Compulsory module

Module units:

- Modelling
- Computer Aided Mathematics
- Numerical Mathematics

Module grade: 2 exams

Learning objectives: This module imparts fundamentals of applied and numerical mathematics. This includes, on the one hand, formulating mathematical models to describe non-mathematical relationships and regularities, as well as practically solving basic mathematical questions from analysis and linear algebra using computers. Since the combination of careful problem description with mathematical analysis of the resulting structure and the development of tailored numerical solution methods is characteristic for projects in applied mathematics, the competencies conveyed here are of great importance for mathematical work in professional practice.

Learning outcomes: The students

- are able to precisely formulate mathematical models for questions from various fields of knowledge and simulate them on the computer using numerical solution methods,
- acquire the ability to read and interpret mathematical models, as well as recognize over- or under-determinations,
- have knowledge of important software packages such as Matlab, Maple, and LaTeX. They are familiar with elementary algorithms for basic numerical tasks and can implement them on the computer,
- can analyze various algorithms for problem solving and evaluate them in terms of computational complexity and the influence of rounding errors (stability).

Module unit Modelling

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorial 1 SWS	2nd semester	annually (summer semester)	German

Recommended prerequisites: Linear algebra I and Analysis I

Teaching contents:

- Constructing a mathematical framework for given problem descriptions in natural sciences,

technology, and economics by identifying relevant variables and specifying their interrelationships in the form of mathematical statements.
- Examples of applications leading to typical mathematical problems (linear and nonlinear equations and inequalities, differential equations, optimization problems, etc.)
- Optimization, probability theory, elements of differential geometry, balance and conservation principles.

Form of examination:
- written or oral exam or term paper
- successful participation in the tutorials

Work load: 135 h
- on-campus studies (lectures and tutorials) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up $14 \times 1,5$ h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- exam preparation: 19,5 h

Module unit Computer aided mathematics

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 1 SWS - tutorial 2 SWS	2nd semester	annually (summer semester)	German

Recommended prerequisites: Linear algebra I and Analysis I

Teaching contents:
- Numerical Computing (number systems, round-off errors, complexity)
- Introduction to mathematical software such as Matlab, Octave, Maple,
- Introduction to

Form of examination:
- written exam or term paper/project work
- successful participation in the tutorials

Work load: 90 h
- on-campus studies (lectures and tutorials) 14×3 h = 42 h
- preparation and follow-up 14×1 h = 14 h
- exercise sheets 14×2 h = 28 h
- exam preparation: 6 h

Module unit Numerical mathematics

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lectures 4 SWS - tutorials 2 SWS	3rd semester	annually (winter semester)	German

Recommended prerequisites: Analysis I/II, Linear algebra I, Computer aided mathematics

Teaching contents: Interpolation, root-finding methods (single and multi-dimensional), linear systems of equations (direct and indirect methods), linear regression, linear optimization, minimization, eigenvalue problems, numerical integration, explicit methods for ordinary differential equations, stability and perturbation issues.

Form of examination:

- exam
- successful participation in the tutorials

Work load: 270 h

- on-campus studies (lectures and tutorials) $14 \times 4{,}5$ h = 63 h
- preparation and follow-up 14×3 h = 42 h
- exercise sheets 14×9 h = 126 h
- exam preparation: 39 h

Compulsory module: Algebra I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	mandatory module

Module grade: written exam

Learning objectives: Familiarity with the fundamental algebraic structures of groups, rings, and fields. These concepts form the foundation for various areas of mathematics. The content of this module is required in all higher-level algebraic or geometric lectures, as well as in modern applications (e.g., coding theory, cryptography) and theoretical physics.

Learning outcomes: The students
- are acquainted with fundamental abstract group, module, and field-theoretical structures and models,
- understand the theory of modules over a ring as a generalization of the theory of vector spaces over a field,
- apply abstract theorems and methods of group, module, and field theory to concrete mathematical problems,
- are capable of analyzing polynomial geometric concepts using abstract algebraic and algorithmic methods,
- can independently prove the main propositions of group, module, and field theory,
- are capable of justifying the correctness of a statement with a proof or refuting it with counterexamples.

Algebra I

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lecture 4 SWS - exercises 2 SWS	3rd semester	annually (winter semester)	German

Recommended prerequisites: Linear algebra I/II

Teaching contents: Fundamentals of commutative rings, group theory, field theory, and Galois theory

Form of examination:
- written exam
- successful participation in the exercises

Work load: 270 h
- on-campus study (lectures and exercises) 14 × 4,5 h = 63 h

- preparation and follow-up 14×3 h = 42 h
- exercises 14×9 h = 126 h
- exam preparation: 39 h

Compulsory module Analysis III

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	mandatory module

Module units:

- Ordinary differential equations
- Measure and integration theory

Module grade: one written exam for both module units

Learning objectives:

- This module provides fundamental knowledge in the theory of ordinary differential equations as well as in general measure and integration theory.
- The students are expected to gain insight into the theory of ordinary differential equations, learn solution methods and abstract approaches to solvability, as well as acquire a modern approach to measure and integration theory (general Lebesgue integral) and be able to apply it.
- This module provides fundamental knowledge for the subsequent advanced modules and is particularly essential for further lectures in the areas of functional analysis, theory of partial differential equations, numerical solutions of ordinary and partial differential equations, and stochastic processes.
- Ordinary differential equations arise in many applications in natural sciences and engineering, as well as in financial mathematics and economics. The knowledge imparted in this module is necessary for a professional career in these fields.

Learning outcomes: The students

- know and understand the fundamental concepts, statements, and methods of the theory of ordinary differential equations,
- have advanced skills in accurately formulating mathematical concepts and logically justifying technical relationships,
- can apply results from analysis and linear algebra to solve problems in the theory of ordinary differential equations,
- understand the significance of ordinary differential equations in various application contexts,
- are capable of employing various solution methods and examining and justifying the qualitative behavior of solutions.

The students

- know and understand the fundamental concepts, statements, and methods of modern measure and integration theory,
- have advanced skills in accurately formulating mathematical concepts and logically justifying technical relationships,
- can utilize the central results of integration theory as tools in solving problems in analysis.

Module unit Ordinary differential equations

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorials 1 SWS (Possibly 4+2 hours during the first half of the lecture period)	3rd semester	annually (winter semester)	German

Recommended prerequisites: Analysis I/II, Linear algebra I

Teaching contents:

- Picard-Lindeloef Existence Theorem
- Uniqueness: Gronwall's Lemma
- Solving Methods for linear systems
- Qualitative aspects: stability
- Solving methods for special equations

Optional topics are:

- Qualitative aspects: Phase Portraits, One-Dimensional Comparison Theorem
- Maximum Flow
- Parameter-Dependent Differential Equations
- Arzelà-Ascoli Theorem, Peano's Existence Theorem
- Boundary and Eigenvalue Problems: Existence and Uniqueness, Green's Function, Eigenvalue Problems

Form of examination:

- Exam in Analysis III
- Successful participation in the exercises

Work load: 135 h

- On-campus studies (lectures and tutorials) $7 \times 4{,}5$ h = 31,5 h
- Preparatino and follow-up 7×3 h = 21 h
- Exercise sheets $7 \times 4,5$ h = 63 h
- Exam preparation: 19,5 h

Module Unit Measure and integration theory

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorial 1 SWS (Possibly 4+2 hours in the second half of the lecture period.)	3rd semester	annually (winter semester)	German

Recommended prerequisites: Analysis I/II, Linear Algebra I

Teaching contents:

- Introduction to General Measure Theory: Measure Spaces, Measurable Functions
- The Lebesgue Integral: Introduction, Convergence Theorems, Product Measure, and Transformation Theorem

Optional topics are:

- L^p-spaces
- Convolution
- Smooth approximation

Form of examination:

- written exam in Analysis III
- successful participation in the tutorials

Work load: 135 h

- On-campus studies (lectures and tutorials) 14 $\times$ 4,5 h = 31,5 h
- Preparation and follow-up 7 $\times$ 3 h = 21 h
- Exercise sheets $14 \times 4,5$ h = 63 h
- Exam preparation: 19,5 h

Compulsory Module Practical mathematics II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Compulsory module with optional module units

Module units:

- Optimization I
  or
- Numerical analysis of ordinary differential equations
  or
- other courses with practical components that are occasionally offered

The event not chosen in the compulsory module Practical mathematics II can still be attended and credited as an elective module.

Module grade: written or oral exam

Learning objectives:

- Following the introductory lectures in the module Practical mathematics I, the focus here is on a more in-depth study of a specific area of numerical mathematics.
- The theme revolves around processes that play a central role in natural sciences, technology, and business. The knowledge gained from the modeling lecture is expanded and deepened accordingly.
- An understanding of the presented numerical solution methods is crucial in many professional fields.
- The exercises cover programming tasks, including more extensive assignments and practical examples to be implemented on the computer, with a focus on providing meaningful documentation of the computational results.
- The compromise between precision and efficiency, crucial for practical work, is conveyed: Typically, the goal is to find a sufficiently good solution in a reasonable amount of time.

Learning outcomes: The students

- are familiar with basic numerical methods in the chosen area and can contextualize central concepts (in optimization, for example, line search, trust region, and (quasi-) Newton methods; in the area of differential equations, concepts like stability, consistency, convergence, and conservation properties)
- can classify the learned methods and, based on the given problem, choose the appropriate method accordingly,
- can implement the derived method on the computer, document the executed steps, and test the correctness of the algorithm,
- can illustrate the theoretical convergence properties of the methods based on the numerical results,
- can justify the chosen solution approach based on theoretical and numerical results,
- can describe interdisciplinary application areas for the considered optimization problems.

Module unit: Optimization I

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - exercises 1 SWS	4th or 6th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Linear algebra I, Numerical mathematics

Teaching contents:

- necessary and sufficient optimality conditions
- descent methods, line search algorithms, convergence analysis
- Newton and quasi-Newton methods
- optimality conditions for constrained problems
- conjugate directions method

Form of examination:

- exam
- successful participation in the exercises

Work load: 135 h

- on-campus study (lecture and exercises) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercises $14 \times 4,5$ h = 63 h
- exam preparation: 19,5 h

Module unit: Numerical analysis of ordinary differential equations

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - exercises 1 SWS	4th semester or 6th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III, Linear algebra I, Numerical mathematics

Teaching contents:

- Introduction to the numerical solution of initial and boundary value problems
- consistency, stability, and convergence
- practical implementation through programming exercises

Form of examination:

- written or oral exam
- successful participation in the exercises

Work load: 135 h

- on-campus study (lectures and exercises) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercises $14 \times 4,5$ h = 63 h
- exam preparation 19,5 h

Compulsory module Probability theory and statistics

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Compulsory module

Module units:

- Probability theory
- Statistics

Module grade: exams

Learning objectives:

- Phenomena subject to chance are ubiquitous. This module provides the fundamental mathematical tools for describing random events, enabling the characterization of the regularities of random processes and deriving them from observations.
- Fundamentals of probability theory, descriptive statistics, inductive statistics.
- The course provides mathematical skills to model stochastic dynamic systems and derive statements from them, allowing for the modeling of real random systems and obtaining insights from them.

Learning outcomes: The students

- can apply basic mathematical tools to describe random processes and characterize and derive some regularities of random processes from observations,
- learn the fundamentals of probability theory as well as descriptive and inductive statistics and can apply them in a differentiated manner,
- can, applying the learned stochastic concepts, obtain and assess results.

Module unit Probability theory

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorial 1 SWS (4+2 hours in the first half of the lecture period)	4th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III (Measure theory), Linear algebra I

Teaching contents:

- Kolmogorov's axioms, independence, conditional probabilities, random variables
- Convergence types in stochastics, characteristic functions
- Law of Large Numbers, Central Limit Theorem.

Form of examination:

- exam
- successful participation in the tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- exam preparation: 19,5 h

Module unit Statistics

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorial 1 SWS	4th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III (Measure theory), Linear algebra I; the second half of the lecture uses contents of the lecture Probability theory, therefore it is recommended to attend both lectures in the same semester

Teaching contents:

- Descriptive statistics: Graphic, tabular, and numerical methods of univariate and multivariate statistics
- Inductive statistics: Important distributions, statistical estimation, confidence intervals, maximum likelihood estimation, statistical testing

Form of examination:

- exam
- successful participation in the tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- exam preparation: 19,5 h

3 Elective modules and areas of specialization

In the Mathematics B.Sc. degree program, one must choose elective modules in such a way that two of the following ares of specialization are covered. At least one of the two covered areas of specialization must be different from Statistics and Stochastics.

Specialization in Analysis und Numerics:

- Elective module Functional analysis
- Elective module Theory and numerics of partial differential equations

Specialization in Differential Geometry:

- Elective module Differential geometry I
- Elective module Differential geometry II
- Elective module Ordinard differential equations with applications to geometry

Spezialization in Geometry and Algebra:

- Elective module Algebra II
- Elective module Algorithmic Algebraic Geometry

Specialization in Statistics:

- Elective module Funtional analysis
- Elective module Mathematical Statistics I

Specialization in Stochastics:

- Elective module Functional analysis
- Elective module Stochastic processes
- Elective module Markov chains

Elective module Algebra II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, specialization in Geometry and Algebra

Module grade: Written or oral exam

Learning objectives: The students gain a solid understanding of module theory and the structure of rings, enabling them to analyze module structures, investigate homomorphisms, and apply the acquired knowledge to various mathematical areas.

Learning outcomes: The students

- are familiar with the basic concepts, definitions, and properties of modules and can apply them to various examples,
- are able to analyze and investigate module structures and can identify various types of modules (such as Noetherian and Artinian modules, free modules, simple, semi-simple, and indecomposable modules),
- have a good understanding of module homomorphisms and their properties, learn the various homomorphism theorems for modules, and can apply them to analyze relationships between modules,
- are familiar with special classes of rings such as Dedekind rings, learn generalizations and variants of unique factorization in factorial rings, such as the prime ideal factorization in Dedekind rings,
- possess sufficient knowledge about modules and rings to apply these concepts in their later studies to various mathematical areas, such as algebraic number theory, algebraic geometry, and representation theory.

Algebra II

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - tutorial 1 SWS (4+2 hours in the first half of the lecture period)	4th semester	annually (summer semester)	German

Recommended prerequisites: Algebra I

Teaching contents:

- Modules, direct sum of modules, free modules, Noetherian and Artinian modules, finitely generated modules over principal ideal rings, Cayley-Hamilton theorem,

- Integral ring extensions and Dedekind rings, integrality, characterization of Dedekind rings

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials)$7 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets 7×9 h = 63 h
- preparation for the exam: 19,5 h

Elective module Algebraic number theory

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module

Module grade: Written or oral exam

Learning objectives: The students are familiar with the arithmetic of integers (also in simple integral domains of algebraic number fields), know the problematics of Diophantine equations, have insights into prime number theory, and know applications of these topics.

Learning outcomes: The students

- are familiar with the basics of algebraic number theory and can reproduce elementary definitions
- are able to outline proofs of central number-theoretical theorems (e.g., finiteness of the class number, Dirichlet's unit theorem) by combining methods and theories from commutative algebra (e.g., Dedekind rings) and geometry (Minkowski theory),
- can analyze concrete elementary number-theoretical questions in the ring of integers and reformulate them into appropriate questions about algebraic number fields, which can then be answered using abstract theorems and methods of algebraic number theory,
- are able to explicitly calculate fundamental numerical invariants of a number field such as discriminant and class number in simple examples, and make predictions about the arithmetic of the number field based on them.

Algebraic number theory

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS (4+2 h in the second part of the semester)	4th or 6th semester	annually (summer semester)	German

Recommended prerequisites: Algebra I, Algebra II

Teaching contents:

- quadratic number fields,
- norm, trace and discriminant,
- ideal class group,
- number rings,
- lattices in number fields,

- finiteness of the class number,
- Dirichlet's unit theorem,
- roots of unity,
- cyclotomic fields.

Form of examination:

- written or oral exam
- successful participation in the exercises and the tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $7 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets 7×9 h = 63 h
- preparation for the exam: 19,5 h

Elective module Algorithmic algebraic geometry

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Elective module, specialisation in Geometry and Algebra

Module grade: Written or oral exam

Learning objectives: The goal is to become familiar with the fundamentals of commutative algebra and algebraic geometry, as well as with the theoretical and practical foundations of their treatment using computer algebra systems. This module serves as a necessary foundation for a deeper exploration (e.g., bachelor's thesis, possibly followed by a master's degree) in the area of the specialization 'Real Geometry and Algebra'. Algorithmics play a significant role in algebraic geometry today, both in theory and applications. The basics of this field will be covered in this module.

Learning outcomes: The students are familiar with the basics of commutative algebra and algebraic geometry and can apply them to fundamental questions arising in the field of computer algebra.

Algorithmic algebraic geometry

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lectures 4 SWS - tutorials 2 SWS	5th semester	annually (winter semester)	German

Recommended prerequisites: Algebra I

Teaching contents:

- Basics of commutative algebra, ideal theory, integrality. Affine and projective varieties, Zariski topology, correspondence between varieties and ideals, regular functions, morphisms, elimination theory,
- Algorithmic treatment of basic ring and ideal operations, Gröbner bases, Buchberger algorithm, working with a suitable computer algebra system.

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 270 h

- on-campus studies (lectures and tutorials) $14 \times 4{,}5$ h = 63 h
- preparation and follow-up 14×3 h = 42 h
- exercises 14×9 h = 126 h

- preparation for the exam: 39 h

Elective module Differential geometry I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, specialisation module Differential Geometry

Module grade: Written or oral exam

Learning objectives:
- Description of curved or nonlinear objects in Euclidean space. Learning the differential geometric calculus.
- This module lays the foundation for later lectures on differential geometry.

Learning outcomes: The students
- understand basic definitions, concepts, statements, and methods of differential geometry,
- recognize the importance of differential geometry for describing surfaces,
- gain a systematic approach to the field.

Differential geometry I

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	4th or 6th semester	irregularly (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III, Linear algebra I/II

Teaching contents: Hyper surfaces, principal curvatures, minimal surfaces, mean curvature flow.

Form of examination:
- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h
- on-campus studies (lectures and tutorials) $7 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets 7×9 h = 63 h
- preparation for the exam: 19,5 h

Elective module Differential geometry II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, specialisation field of Differential Geometry

Module grade: Written or oral exam

Learning objectives:
- Description of curved or nonlinear objects in Euclidean space. Deepening of the differential geometric calculus.
- Utilization of numerous differential geometric definitions in calculations and proofs.

Learning outcomes:
- The students expand their description capabilities for submanifolds and apply them in various ways.
- The students can infer global properties from local ones.
- The students master more elaborate proofs in the field.

Differential geometry II

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	6th semester	Irregularly (summer semester)	German

Recommended prerequisites: Differential geometry I

Teaching contents: Curves, isoperimetric inequality, Fenchel's theorem, distance function, mean curvature flow, ordinary differential equations on manifolds, geodesics.

Form of examination:
- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h
- on-campus studies (lectures and tutorials) $7 \times 4{,}5$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets 7×9 h = 63 h
- preparation for the exam: 19,5 h

Elective module Functional analysis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, specializations in Analysis and Numerics, Statistics and Stochastic

Module grade: Written or oral exam

Learning objectives:

- Acquisition of basic knowledge about mappings between general metric and normed spaces, which are particularly essential in Analysis and Numerics.
- Provision of fundamental basics for further lectures in the areas of Analysis and Numerics, especially for lectures on partial differential equations.
- Functional-analytical methods are to be learned and applied, with a focus on abstract approaches to concrete questions (such as differential equations). The overarching goal is to recognize abstraction as a crucial tool for simplification and clarity.
- The ability to abstract is a crucial skill of a mathematician in the profession and sets them apart. Therefore, this module is highly relevant for the profession, with methodology being more important than specific knowledge.

Learning outcomes: The students

- know and understand basic terms, statements, and methods of functional analysis,
- recognize the significance of functional analysis for studies in analysis and numerics,
- are familiar with abstract approaches to general questions. Knowledge acquired in the mandatory modules Analysis I/II and Linear Algebra I/II (such as the theory of Fourier series and the spectral theorem for matrices) is now reconsidered from a more abstract and general standpoint.
- recognize abstraction as a crucial tool for simplification and clarity, which is also an outstandingly relevant professional competence.

Functional analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorial 1 SWS	4th or 6th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III, Linear algebra I/II, also useful is Complex analysis

Teaching contents:

- Normed spaces, linear mappings in normed spaces

- Hahn-Banach theorem
- Hilbert spaces
- orthogonality
- dual space, Blaire's theorem and its consequences
- optional: topological foundations, completion, L^p spaces, projection onto convex sets

The topics covered in Functional Analysis or Functional Analysis II include:

- reflexivity
- weak convergence

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) 14 $\times$ 2,25 h = 31,5 h
- preparation and follow-up 14 $\times$ $1,5$ h = 21 h
- exercise sheets 14 $\times$ $4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Funktional analysis II

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module

Module grade: Written or oral exam

Learning objectives:
- The goal is to acquire basic knowledge in the theory of mappings between general metric and normed spaces, which are essential especially for studies in Analysis and Numerics.
- The knowledge imparted in this module is fundamental for lectures in the areas of Analysis and Numerics, especially for lectures on partial differential equations in Bachelor's and Master's programs.
- Functional-analytical methods are to be learned and applied, with a focus on abstract approaches to concrete questions (such as differential equations). The overarching goal is to recognize abstraction as a crucial tool for simplification and clarity.
- The ability to abstract is a crucial skill of a mathematician in the profession and sets them apart. Therefore, this module is highly relevant for the profession, with methodology being more important than specific knowledge.

Learning outcomes: The students
- know and understand the fundamental terms, statements, and methods of functional analysis,
- recognize the significance of functional analysis for studies in Analysis and Numerics,
- are familiar with abstract approaches to general questions. Knowledge acquired in the mandatory modules Analysis I/II and Linear Algebra I/II (such as the theory of Fourier series and the spectral theorem for matrices) is now reconsidered from a more abstract and general standpoint,
- recognize abstraction as a crucial tool for simplification and clarity, which is also an outstandingly relevant professional competence.

Functional analysis II

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	6th semester	occasionally (summer semester)	German

Recommended prerequisites: Functional analysis, helpful is furthermore Complex analysis

Teaching contents:

Sobolev spaces or operators are discussed in more detail, while the other topic is presented in an overview.

- Sobolev spaces
- closed operators
- spectrum of operators, spectral theorem for self-adjoint mappings
- optional: distributions

Topics that are discussed in Functional Analysis or Functional Analysis II:

- reflexivity
- weak convergence

Form of examination:

- Written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up $14 \times 1,5$ h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Complex analysis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module

Module grade: Written or oral exam

Learning objectives: Learning characteristic properties of functions of a complex variable and the use of special methods as tools. This module is fundamental for many areas of mathematics.

Learning outcomes: The students

- know and understand basic terms, statements, and methods of function theory,
- possess advanced skills in precisely formulating mathematical concepts and logically justifying technical relationships,
- understand how function theory enables a deeper understanding of results in real analysis and contributes to central results in algebra.

Complex analysis

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	4th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Linear Algebra I/II

Teaching contents:

- complex differentiability
- Cauchy-Riemann differential equations
- Cauchy's integral formula (different variants),
- Liouville's theorem,
- Fundamental theorem of algebra,
- representation as power series,
- Morera's theorem,
- Reflection principle,
- simply connected domains,
- existence of a primitive function,
- isolated singularities,
- Residue theorem with applications to integrals.

Optional content includes:

- oppen mapping theorem

- conformal mappings and Riemann mapping theorem.

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up $14 \times 1,5$ h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Ordinary differential equations with applications to geometry

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, specialisation field of Differential Geometry

Module grade: Written or oral exam

Learning objectives: Mathematical description of curved objects

Learning outcomes: The students
- can describe submanifolds,
- can express geometric questions using differential equations,
- can investigate properties of solutions to the occurring nonlinear ordinary differential equations, especially in the case of first-order equations.

Ordinary differential equations with applications to geometry

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	5th semester	occasionally (winter semester)	German

Recommended prerequisites: Analysis I/II, Analysis III, Linear algebra I/II, differential Geometry I

Teaching contents:
- Knowledge about embedded manifolds and the ability to investigate their properties by studying ordinary differential equations.
- Exemplary: Investigation of rotationally symmetric translating solutions of mean curvature flow.

Form of examination:
- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h
- on-campus studies (lectures and tutorials) 14 $\times$ 2,25 h = 31,5 h
- preparation and follow-up $14 \times 1,5$ h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Markov chains

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective Module, field of specialisation Stochastics

Module grade: Written or oral exam

Learning objectives: The lecture provides an overview of the theory of Markov chains in discrete and continuous time, which are encountered in many applications and can be viewed as simple prototypes of general Markov processes. It turns out that determining the distribution of certain objects can be reduced to solving certain deterministic equations (linear systems of equations, ordinary differential equations). This provides an initial example of the interplay between probabilistic and analytical methods.

Learning outcomes: The students
- can recognize situations that can be described by Markov chains and in concrete cases formulate an appropriate model,
- are familiar with the fundamental concepts and theorems of the theory of Markov chains,
- can use methods from linear algebra and ordinary differential equations to calculate distributions of certain random variables,
- recognize interconnections between stochastic processes and analysis.

Markov chains

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS	5th semester	annually (winter semester)	German

Recommended prerequisites: Analysis I-III, Probability theory, Stochastic processes

Teaching contents:
- the Poisson process
- continuous-time Markov chains: intensity matrix and transition semi-groups
- Potential theory: harmonic functions, Green's matrix, Dirichlet problem
- recurrence and transience, invariant distributions, asymptotic behavior

Form of examination:
- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) 14 $\times$ 2,25 h = 31,5 h
- preparation and follow-up 14 $\times$ $1,5$ h = 21 h
- exercise sheets 14 $\times$ $4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Mathematical statistics I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Elective module, specialization in Statistics

Module grade: Written or oral exam

Learning objectives: Students gain an overview of situations in which uniformly most powerful (unbiased) tests exist. They are familiar with basic statements about the asymptotics of maximum likelihood estimators and likelihood ratio tests.

Learning outcomes: The students can

- construct optimal tests,
- handle convergence in distribution,
- construct asymptotically efficient estimators using the maximum likelihood method,
- derive asymptotic tests from the likelihood ratio approach.

Mathematical statistics I

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lectures 4 SWS - tutorials 2 SWS	5th semester	annually (winter semester)	German

Recommended prerequisites: Probability theory and statistics

Teaching contents:

- Uniformly most powerful tests and uniformly most powerful unbiased tests for standard hypotheses, tests with nuisance parameters
- Exponential families
- Convergence in distribution, delta method, order statistics
- Existence and asymptotic normality of maximum likelihood estimators
- Maximum likelihood estimators in exponential families
- Asymptotics of likelihood ratio tests

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up $14 \times 1,5$ h = 21 h

- exercise sheets $14 \times 4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Numerical analysis of ordinary differential equations

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module

Module grade: Written or oral exam

Learning objectives: The brief introduction to the numerical solution of ordinary differential equations from the Numerical Mathematics course is continued and deepened. In addition to theoretical questions regarding the consistency and stability of approximation methods, the practical aspects involving algorithms and programming tasks become more extensive. This course serves as a fundamental basis for further lectures on the numerical solution of differential equations in the Änalysis and Numerics"focus area. Since ordinary differential equations play a significant role in modeling dynamic processes in natural sciences, technology, and economics, understanding corresponding numerical solution methods is essential in many professional fields.

Learning outcomes: The students
- are familiar with basic numerical methods for solving ordinary differential equations and can contextualize the concepts of stability, consistency, and convergence,
- are able to classify the learned methods and select the appropriate technique for various initial and boundary value problems,
- can implement the derived method on a computer, document the steps taken, and verify the correctness of the algorithm,
- are able to assess the numerical results concerning the given initial or boundary value problem and illustrate the theoretical properties such as stability, consistency, and convergence based on these results,
- can justify the chosen solution approach based on the theoretical and numerical results.

Module unit: Numerical analysis of ordinary differential equations

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - exercises 1 SWS	4th semester or 6th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Analysis III, Linear algebra I, Numerical mathematics

Teaching contents:
- Introduction to the numerical solution of initial and boundary value problems
- consistency, stability, and convergence
- practical implementation through programming exercises

Form of examination:

- written or oral exam
- successful participation in the exercises

Work load: 135 h
- on-campus study (lectures and exercises) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercises $14 \times 4,5$ h = 63 h
- exam preparation 19,5 h

Elective module Optimization I

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module

Module grade: Written or oral exam

Learning objectives: The main goal of the lecture is to provide fundamental knowledge about the theory of non-linear optimization problems, numerical methods, and their practical implementation using computer-based examples. A meta-goal is to understand that often it's not crucial to determine the exact optimum, but rather to find a sufficiently good solution in a reasonable amount of time.

Learning outcomes: The students
- understand basic numerical methods for unconstrained optimization and can categorize concepts such as line search, trust region, and (quasi-)Newton methods,
- are able to classify the learned methods and select the appropriate method according to the given optimization problem,
- can implement the derived method on the computer, document the executed steps, and test the correctness of the algorithm,
- can verify, based on optimality conditions, whether an optimal numerical solution has been obtained and illustrate the theoretical convergence properties of the optimization methods based on the numerical results,
- are able to justify the chosen solution path based on the theoretical and numerical results,
- can describe interdisciplinary application areas for optimization problems.

Module unit: Optimization I

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lecture 2 SWS - exercises 1 SWS	4th or 6th semester	annually (summer semester)	German

Recommended prerequisites: Analysis I/II, Linear algebra I, Numerical mathematics

Teaching contents:
- necessary and sufficient optimality conditions
- descent methods, line search algorithms, convergence analysis
- Newton and quasi-Newton methods
- optimality conditions for constrained problems
- conjugate directions method

Form of examination:

- exam
- successful participation in the exercises

Work load: 135 h

- on-campus study (lecture and exercises) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercises $14 \times 4,5$ h = 63 h
- exam preparation: 19,5 h

Elective module Stochastic processes

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	3	Elective module, field of Stochastics

Module grade: Written or oral exam

Learning objectives: The lecture provides an introduction to the theory of stochastic processes in discrete and continuous time. Classes of stochastic processes are introduced that are of utmost importance for both theory and applications. Specifically, Markov chains, martingales, and Brownian motion are presented.

Learning outcomes: The students
- can model random dynamic processes using stochastic processes and apply the essential regularities differentially,
- master the mathematical foundations for the analysis of stochastic financial market models.

Stochastic processes

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	- lectures 2 SWS - tutorials 1 SWS (4+2 h in the second half of the semester)	4th semester	annually (summer semester)	German

Recommended prerequisites: Module unit Probability theory from the mandatory module Probability theory and statistics (taught in the first half of the lecture period in the summer semester and can be completed in the same semester).

Teaching contents:
- Conditional expectations
- Markov chains: recurrence and transience
- Martingales: Doob's inequalities, optional stopping, Martingale convergence theorem
- Construction of Brownian motion.

Form of examination:
- written or oral exam
- successful participation in the exercises and tutorials

Work load: 135 h

- on-campus studies (lectures and tutorials) $14 \times 2{,}25$ h = 31,5 h
- preparation and follow-up 7×3 h = 21 h
- exercise sheets $14 \times 4,5$ h = 63 h
- preparation for the exam: 19,5 h

Elective module Theory and numerics of partial differential equations

ECTS	Duration	SWS (Weekly teaching hours)	Placement
9	1 semester	6	Elective module, field of Analysis and Numerics

Module grade: Written or oral exam

Learning objectives: The lecture provides an overview of theoretical and practical aspects of partial differential equations. The focus is on the classification of the main types and their treatment using analytical-theoretical and numerical methods, as well as their respective relevance in applications. Students deepen their programming skills and become familiar with software packages in the field of finite elements.

Learning outcomes: The students
- understand and comprehend fundamental concepts, statements, and methods in the theory and numerical treatment of partial differential equations,
- can apply methods of analysis, especially functional analysis, to problems involving partial differential equations,
- recognize the connection between theory and numerical methods and the significance of the subject for applications.

Theory and numerics of partial differential equations

ECTS	Teaching methods	Recommended semester	Frequency	Language
9	- lectures 4 SWS - tutorials 2 SWS	5th semester	annually (winter semester)	German

Recommended prerequisites: Analysis I-III, Linear algebra I/II, Numerical mathematics, Functional analysis

Teaching contents:
- linear partial differential equations (PDEs) of first order
- classification of second-order partial differential equations
- elliptic partial differential equations (Perron's method), hyperbolic partial differential equations (separation of variables), parabolic partial differential equations (classical solutions, maximum principle)
- Hilbert space methods for elliptic, hyperbolic, and parabolic partial differential equations
- finite difference methods for elliptic boundary value problems
- difference methods for parabolic problems, line method
- conservative methods for hyperbolic conservation equations
- introduction to the Finite Element Method

- consistency, stability, convergence

Form of examination:

- written or oral exam
- successful participation in the exercises and tutorials

Work load: 270 h

- on-campus studies (lectures and tutorials) $14 \times 4{,}5$ h = 63 h
- preparation and follow-up 14×3 h = 42 h
- exercise sheets 14×9 h = 126 h
- preparation for the exam: 39 h

Irregularly offered elective modules

In addition to the elective modules listed above, additional elective modules are offered, which are not offered regularly. A module description will then be announced by the respective lecturer.

In recent years, the following elective modules have been offered, among others:

- Algorithmic number theory
- Axiomatic set theory
- Computability, Turing machines
- Valuation theory
- Representation theory of finite groups
- Representation theory and invariant theory of finite groups
- Differential geometry
- Dynamical systems
- Fourier analysis of Boolean functions
- Fourier transform and Sobolev spaces
- Fundamental groups
- Inverse problems
- Combinatorial optimization
- Commutative algebra
- Mathematical foundations of quantum mechanics
- Mathematical logic
- Model theory
- Non-negative matrices
- Numerics of stochastic differential equations
- O-minimal geometry
- Polynomial optimization
- Prime numbers in theory and practice
- Quadratic forms
- Recursion theory
- Recursive functions, Turing machines
- Ring theory
- Spectral theory
- Stability of nonlinear waves
- Topology
- Topological vector spaces
- Calculus of variations
- Generalized linear models
- Actuarial mathematics

4 Seminars und bachelor’s thesis

Introductory seminar course („Proseminar“)

ECTS	Duration	SWS (Weekly teaching hours)	Placement
3	1 semester	2	Mandatory seminar

Module grade: Not graded or according to the evaluation of the oral presentation

Learning objectives: According to the announcements by the lecturer

Learning outcomes: Participants are generally expected to be able to present relatively simple mathematical concepts with a full understanding. A presentation serves as an introduction to the basic of information literacy. This includes presenting independently acquired mathematical concepts and conveying them to an audience of fellow students.

Introductory seminar course („Proseminar“)

ECTS	Teaching methods	Recommended semester	Frequency	Language
3	2 SWS	3rd or 4th semester	each semester	German

Recommended prerequisites: Analysis I/II and Linear algebra I/II

Teaching contents: According to the announcements by the lecturer

Form of examination:

- oral presentation
- presence and active participation
- possibly a written paper

Work load:

- 20 h on-campus studies
- 70 h self-studies

Advanced Seminar

ECTS	Duration	SWS (Weekly teaching hours)	Placement
4,5	1 semester	2	compulsory seminar

Module grade: not graded or according to the assessment of the oral presentation

Learning objectives: as announced by the instructor

Learning outcomes: Independent scientific work using a clearly defined topic as an example. Participants should be enabled to work on a subject, present it comprehensibly, and express it appropriately in writing. The careful handling of literature, including independent and efficient research, presentation techniques using appropriate presentation media, and academic writing are conveyed and practiced further

Advanced seminar

ECTS	Teaching methods	Recommended semester	Frequency	Language
4,5	2 SWS	5th semester	annually	German

Recommended prerequisites: as announced by the instructor

Teaching contents: as announced by the instructor

Form of examination:

- Oral presentation
- Presence and active participation
- possibly a written term paper

Work load:

- 20 h on-campus studies
- 115 h self-study

Bachelor's Thesis

ECTS	Duration	SWS (Weekly teaching hours)	Placement
15	4 months	0	mandatory

Module grade: determined based on the evaluation of the work by a professor or lecturer from the department

Learning objectives: based on the topics provided by the instructors

Learning outcomes: The bachelor's thesis is intended to demonstrate that the candidate is capable of proficiently addressing a more extensive task in the field of mathematics and applying mathematical methods appropriately.

Bachelor's Thesis

ECTS	Teaching methods	Recommended semester	Frequency	Language
15	0 SWS	6th semester	annually	German

Recommended prerequisites: completion of the modules within a specialization area from which the thesis is supposed to emerge

Teaching contents: determined by the topics provided by the lecturer

Form of examination:

- Bachelor's thesis
- Presentation for the bachelor's thesis

Work load:

- 30 h personal supervision
- 420 h self-study